Ergodic Theory Seminar

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The following is the schedule for the talks to be given in the Ergodic Theory seminar. All references are to the main text for the course, *Ergodic Theory: Independence and Dichotomies* by Kerr and Li.

List of talks

Talk 1: Foundations and examples

Date: 14/10/2025
Speaker: Elisabeth Chanduvi Suarez

In this talk, we introduce some foundational concepts used in ergodic theory. Standard probability spaces (§1.4) and p.m.p. actions (§1.5) are defined (see also Appendix A), as well as natural notions such as conjugacy and factor maps. Basic terminology of Hilbert space operators and unitary representations is reviewed (§1.8) in order to define the Koopman representation (§1.9). Finally, the definitions are illustrated through several examples: odometers, Bernoulli shifts and rotations (§2.3).

Talk 2: Ergodicity, freeness and Poincaré recurrence

Date: 21/10/2025
Speaker: Markus Meyer zu Westrup

We motivate and define ergodicity (Definition 2.1) and freeness (Definition 2.2) for p.m.p. actions. Equivalent characterizations are given (Propositions 2.4, 2.5 and 2.7) before proving the Poincaré recurrence theorem (Theorem 2.10). We illustrate these concepts via the examples introduced in Talk 1. For instance, we show that rotations are ergodic if and only if the angle of rotation is irrational, and determine which of these actions is free. We discuss with examples why Poincaré recurrence may fail for infinite measure spaces.

Talk 3: Amenability and averaging

Date: 28/10/2025
Speaker: Jin Jun Liu

In this talk, we take a short detour from dynamics to discuss amenability of groups. We give the basic definition (Definition 4.1) and provide an alternative formulation in terms of Følner sequences (Theorem 4.4). We provide basic examples of amenable and non-amenable groups (Examples 4.5 and 4.7). Finally, we briefly sketch the proof of Proposition D.17, which provides a canonical means of averaging for amenable groups and underpins the statement of the ergodic theorems in Talks 4 and 5, and the definition of weak mixing for p.m.p. actions of amenable groups we encounter in Talk 6.

Talk 4: Mean ergodic theorem

Date: 04/11/2025
Speaker: Pascal Schreiber

First proved for integer actions by von Neumann, the mean ergodic theorem is one of the first major results in ergodic theory. In this talk, we discuss the ergodicity hypothesis and present a proof of the mean ergodic theorem for actions of amenable groups—a result first proved in this generality by Dye (Theorem 4.22 and 4.23). As one consequence, we obtain a new characterization of ergodicity for p.m.p. actions (Theorem 4.21). We also mention the abstract Hilbert space version of the mean ergodic theorem (Theorem 2.21) and, in preparation for Talk 5, introduce tempered Følner sequences (Definition 4.26).

Talk 5: Pointwise ergodic theorem

Date: 11/11/2025
Speaker: Hyeseo Lee

We prove the pointwise ergodic theorem for actions of amenable groups (Theorem 4.28), which upgrades the L^1 convergence of the mean ergodic theorem to pointwise convergence. This was first proved by Birkhoff for the integers and by Lindenstrauss for amenable groups. The key step is the maximal inequality (Theorem 4.27). Time permitting, we present as an application Borel's theorem on normal numbers.[1]

Talk 6: Mixing and compactness

Date: 18/11/2025
Speaker: Antareep Saud

We introduce (strong) mixing and weak mixing for p.m.p. actions (Definitions 2.11 and 2.15; see also Theorem 4.21), along with the Hilbert space formulation of mixing (Proposition 2.14), and some basic consequences of weak mixing (Proposition 2.16). We define compactness for unitary representations and p.m.p. actions (Definitions 2.22 and 2.26) and prove Theorem 2.28—a relative version of which is the basis for the structure theorem discussed in Talk 7. Finally, we prove that Bernoulli shifts are mixing and that odometers and rotations are compact (§2.3).

[1] For a proof, see Theorem 1.15 of Peter Walters' *An Introduction to Ergodic Theory*.

Talk 7: Furstenberg–Zimmer structure theorem

Date: 25/11/2025
Speaker: Marvin Blankenstein

The Furstenberg–Zimmer structure theorem is a powerful general result that reduces the complexity of answering many questions in ergodic theory. For instance, as we will see in Talk 8, it is the key to Furstenberg's proof of Szemerédi's theorem. In this talk, we recall some relevant terminology about Hilbert modules (§3.1) and formulate relative/conditional versions of weak mixing and compactness (Definition 3.8). Using these, we outline the proof of the structure theorem (Theorem 3.15). Key intermediate steps are Lemmas 3.11 and 3.14.

Talk 8: From multiple recurrence to Szemerédi's theorem

Date: 02/12/2025
Speaker: Vishal Chawla

In this talk, we reach a milestone of the seminar: Szemerédi's theorem (Theorem 3.26). A result in number theory first proved by Szemerédi by combinatorial methods, we present Furstenberg's surprising proof using his multiple recurrence theorem (Theorem 3.25). We sketch the outline for the proof and explain the correspondence between Szemerédi's theorem and Furstenberg's multiple recurrence theorem. Time permitting, we present some parts of the proof in detail (§3.3).

Talk 9: Orbit equivalence and Rokhlin's lemma

Date: 09/12/2025
Speaker: Sonja Wiewrodt

In this talk, we begin our exploration of a different facet of ergodic theory, namely orbit equivalence. We explain how every p.m.p. action gives rise to an orbit equivalence relation (Example 4.54), which motivates the definition of orbit equivalence (Definition 4.76). We mention (without proof) the Feldman–Moore theorem which establises the converse statement. We prove the Rokhlin lemma (Lemma 4.77) and briefly discuss its generalization to actions of amenable groups, the Ornstein–Weiss quasitower theorem (Theorem 4.44).

Talk 10: Dye's theorem

Date: 16/12/2025
Speaker: Sam Shepherd

We continue our discussion of orbit equivalence by proving Dye's theorem (Theorem 4.83), a foundational result in the theory of orbit equivalence stating that any two ergodic integer actions on atomless probability spaces are orbit equivalent. The proof is one of many applications of Rokhlin's lemma.

Talk 11: Connes–Feldman–Weiss and Ornstein–Weiss theorems

Date: 13/01/2026
Speaker: Robin Sroka

In this talk we prove the Ornstein–Weiss theorem (Theorem 4.84), which establishes that there is a unique orbit equivalence class for p.m.p. actions of (infinite) amenable groups on atomless probability spaces. By proving the Connes–Feldman–Weiss theorem (Theorem 4.72), we reduce the problem to Dye's theorem as proved in Talk 10.

Talk 12: Entropy rigidity for Bernoulli shifts

Date: 20/01/2026
Speaker: Eduardo Silva

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